

# FORMAL ENHANCED CATEGORY THEORY

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Whenever one deals with mathematical structures with a hierarchy of morphisms between them, depending how tightly or loosely they preserve the structure, one is led to an enhanced category context. The purpose of the talk is to show that the enhanced 2-categories introduced by Lack and Shulman [6] form an enhanced 2-category  $Enh2Cat$  whose tight (loose) 1-cells are enhanced (lax) 2-functors and 2-cells are tight enhanced oplax transformations. This fact allows us to investigate formal properties of enhanced 2-categories along the lines of Gray [3] which provide a natural (and in some universal sense a minimal) framework in which it is possible to organize both lax functors and oplax transformations into a 2-category. The 2-category  $Enh2Cat$  is enriched, in the sense of [2] in the 2-category  $\mathbb{F}$  whose objects are full embeddings. The major class of examples are given by Eilenberg-Moore enhanced 2-categories  $T - Alg$  of algebras over an enhanced 2-monad, whose tight and loose morphisms are strict and lax  $T$ -algebra morphisms respectively. Another major example is the enhanced 2-category  $Fib$  of fibrations, whose tight and loose 1-cells are cartesian and oplax functors between them, respectively. This example generalizes to generalized fibrations à la Benabou, with his definition of a cartesian functor between general fibrations (or any functors). The enhanced context is particularly suitable for Johnstone's characterization of internal fibrations in 2-categories and bicategories. Bunge and Funk's bicomma object condition for KZ-doctrines seems rather technical and it defines admissible KZ-monads which are precisely the class of  $KZ$ -monads whose Eilenberg-Moore enhanced 2-category  $T - Alg$  consists exactly of cocomplete objects and cocontinuous morphisms between them. It has a beautiful and simple interpretation in the context of enhanced 2-fibrations.

## REFERENCES

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