Homotopy setoids and quotient completion

Cipriano Junior Cioffo

Università degli Studi di Milano

Setoids are a concept of constructive mathematics, originally introduced by Bishop [1], which can be properly described in Martin-Löf intuitionistic type theory [2], through closed types equipped with equivalence relations. The category **Std** of setoids and functions preserving relations has been widely studied, and turns out to be a pretopos [3] obtained as the exact completion of the category of closed types.

We have considered a homotopical version of setoids (h-setoids) in view of ideas from the homotopy type theory [4]. Unfortunately, h-setoids form a category **h-Std** which is not exact in the sense of Barr [5]. Hence, elementary doctrines [6] provide a fruitful framework to study the category **h-Std**, which appears as an instance of a more general quotient completion, namely the elementary quotient completion.

In this talk, we will present some of the properties we obtained for the category **h-Std,** such as the local cartesian closure. Moreover, we will also discuss how this work led to new considerations on the role of strict products in categorical logic.

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