Context, judgement, deduction

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The notions of context, judgement, and deduction all pertain to logic and are in fact the basic blocks which most traditions in the subject are built on. While mathematicians rightly feel they have a grasp on what each of these pieces are, and how to deal with them, the perspective which one comes from decisively skews the way they interpret each: for example, a string of symbols as the following

$$\heartsuit \vdash \clubsuit$$

would be interpreted *as a judgement* by someone studying dependent type theory, so that $\clubsuit$ is a type in context $\heartsuit$, or *as a consecution* if the person is a proof theorist, with $\clubsuit$ being something we can derive from $\heartsuit$. Still, there is no reason as to why these two perspectives should not be supported by the same theoretical system, and this is the topic of the work we will be presenting. In fact, such an effort has already been attempted and proved fruitful. We just take it to the extreme, and see how that brings remarkable results.

We begin breaking up what syntactic data is needed in order to give an account of context, judgement, and deduction, and to each we assign a 2-categorical notion. We then show how to generate complex data (what we call *nested judgements*), and provide a few examples of the computational power that this yields. With the tools we have built, we go to present a notion of "judgemental dependent type theory", and see how it fits with other models from the literature.

We show how our efforts prove to be interesting in at least three different regards: they make it possible to give definitions that could not be grasped before, such as that of (a generic) type constructor, featuring the usual formation, introduction, elimination, and computation rules; they yield a calculus that fits different deduction systems; of these deduction systems, they allow for (and in fact, encourage) a comparative analysis. With respect to this, we conclude providing an insight connecting the cut rule from proof theory and that regulating type-dependency in dependent type theory.

This is joint work with Ivan Di Liberti.